[optimal irrigation plans for two local farms]

Solving Problems 2.3 and 7.1 Using the Dual Formation and Mixed Integer Methods

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# Farm 1: Hay and Grain Crop Allocations

An aqueduct supplying industrial interests has an excess capacity in June, July, and August of 14,000, 18,000, and 6,000 acre-ft of water, respectively. The excess water is valuable for developing irrigated farming on nearby land. The farm owner stated not more than 10,000 acres of land will be allocated to crops, and irrigation requirements vary monthly (Bishop, Hughes and McKee 1999, 36). The report for Farm 1 details two complementary linear programming models, called the primal and dual formations, used to find an optimal planting and irrigation plan for the farm owner. The solutions and insights gained from each formation are also discussed.

# Methods

The primal optimizes the production of resources to produce maximum benefits (or minimize disbenefits), and the dual optimizes the the unit values (or cost) of resources to minimize costs. The constraints in the primal form are the decisions in the dual form, and vice versa (Rosenberg, Lecture: Dual Formation 2020). Detailed model formations are shown in Appendix A: Problem 2.3 Primal and Dual Formulations. Both the primal and dual models for this irrigation problem were coded and analyzed using General Algebraic Modeling Software (GAMS). The model code and outputs are available in the author’s [GitHub repository](https://github.com/joshuatward/JTW_CEE5410_Repo/tree/master/HW5%20Dual%20and%20Integer) (Ward 2020).

# Results

The primal and dual model solutions are shown below:

**Primal Formulation**

**Net Benefits**: $1.16 million

Table 1. Primal Decision Variable Values

|  |  |
| --- | --- |
| **Variable** | **Acreage** |
| Hay | 2000 |
| Grain | 8000 |

Table 2. Primal Constraint Levels & Marginal Values

|  |  |  |
| --- | --- | --- |
| **Constraint** | **Level** | **Marginal ($)** |
| Land | 10,000 | 80 |
| Jun water | 12,000 | 0 |
| Jul water | 18,000 | 20 |
| Aug water | 2,000 | 0 |

**Dual Formulation**

**Reduced Cost**: $1.16 million

Table 3. Dual Decision Variable Values

|  |  |  |
| --- | --- | --- |
| **Variable** | **$/Unit** | **Marginal** |
| Land | 80 | 0 |
| Jun water | 0 | 2000 |
| Jul water | 20 | 0 |
| Aug water | 0 | 4000 |

Table 4. Dual Constraint Levels & Marginal Values

|  |  |  |
| --- | --- | --- |
| **Constraint** | **$/acre** | **Marginal (acres)** |
| Hay | 100 | 2,000 |
| Grain | 120 | 8,000 |

# Comparison and Conclusion

The primal and dual solutions calculate $1.16 million in net benefits/reduced cost. The primal solution more directly interprets the amount of resources used (Table 1, Table 2), whereas the dual solution more directly gives the unit price of the constrained resources (Table 3; see marginal of Table 2). The primal solution is also easier to grasp conceptually than the dual. The dual form also offers two more insights:

* The marginal values on the dual decision values indicate an additional 2000 and 4000 ac-ft of water must be used to create a unit value in June and August water, respectively.
* The marginal values on the dual constraint levels indicate that increasing the primal objective function coefficients by $1/acre is worth planting an additional 2,000 and 8,000 tomato plants, respectively, to maximize total benefits.

# Farm 2: Building a Reservoir and Pumping from a River

“To build, or not to build? That is [a] question” facing a second farmer, who wants to develop water in a channel for crop irrigation. To provide the needed water, a reservoir could be built to dam the river, and a pump could be installed at a lower reach of the channel to capture groundwater drainage. The farmer is interested in which decisions will lead to the most overall profit over two growing seasons, each with different water availability and irrigation demand (Bishop, Hughes and McKee 1999). This section discusses a mixed integer programming model used to locate an optimal solution for the farmer. It is recommended that the farmer construct a high capacity reservoir and install a pump at a lower reach of the channel to irrigate 382.5 acres of crops.

# Methods

This irrigation model was coded and analyzed using General Algebraic Modeling Software (GAMS). The model code and outputs are available in the author’s [GitHub repository](https://github.com/joshuatward/JTW_CEE5410_Repo/tree/master/HW5%20Dual%20and%20Integer) (Ward 2020).

The mixed integer model contains both continuous and binary variables. The farmer must decide how many acres to irrigate (continuous), when and where to divert water (continuous), and which infrastructure to install to provide the needed water (binary; build, or don’t build). The model objective is to maximize total profits within the constraints of the system, which include the following:

* Reservoir capacity, depending on the reservoir choice.
* Irrigation demand of the crops.
* Pump capacity, which limits water delivery via a pump.
* The farmer may build a maximum of one reservoir and a maximum of one pump
* The inflows and outflows of the system must be balanced at the reservoir and pump sites.

Appendix B: Problem 7.1 Formulation contains a more detailed formulation of the model.

# Results and Recommendation

The model results indicate a maximum profit of $82,150 dollars annually if the farmer builds a high capacity (700 ac-ft) reservoir, installs a pump at the lower reach of the river, and follows the recommended allocations in Table 5 to irrigate 382.5 acres of crops. Every ac-ft of water the farmer leaves in the stream will reduce profits by $20/ac-ft of water, and every ac-ft of water the farmer leaves in the reservoir during the second season will reduce profits by $75/ac-ft of water.

Table 5. Recommended water allocations in each season.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Location** | **Season 1 (First 6 months)** | | **Season 2 (Second 6 months)** | |
| **Water (ac-ft)** | **Marginal ($)** | **Water (ac-ft)** | **Marginal ($)** |
| Reservoir Storage | 582.5 | 0 | 0 | -75 |
| Diverted from Reservoir to farm | 17.5 | 0 | 782.5 | 0 |
| Streamflow | 0 | -20 | 0 | -20 |
| Pumped from river | 365 | 0 | 365 | 0 |

# References

Bishop, A Bruce, Trevor Hughes, and Mac McKee. 1999. *Water Resources Systems Analysis - Course Notes.* Logan: Utah State University. https://digitalcommons.usu.edu/ecstatic\_all/76/.

Rosenberg, David E. 2020. "Lecture: Dual Formation." *CEE 5410: Water Resource Systems Analysis.* Logan: Utah State University, September 22.

—. 2020. "Lecture: In-class Integer Programming Model Formulation." *CEE 5410: Water Resource Systems Analysis.* Logan: Utah State University, October 02.

Ward, Joshua Timothy. 2020. "JTW\_CEE5410\_Repo/HW5 Dual and Integer/." *GitHUb.* October 05. Accessed October 05, 2020. https://github.com/joshuatward/JTW\_CEE5410\_Repo/tree/master/HW5%20Dual%20and%20Integer.

# Appendix A: Problem 2.3 Primal and Dual Formulations

Table 6. Monthly water requirements (acre-ft) and return per acre planted

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Crop | June | July | August | Return, $/acre |
| Hay | 2 | 1 | 1 | 100 |
| Grain | 1 | 2 | 0 | 120 |

**Primal Formulation**

**Decision Variables**

* AH: Acres of hay to plant
* AG: Acres of grain to plant

**Constraints**

* Total land area cannot exceed 10,000 acres.
* Total water used for irrigation in June cannot exceed 14,000 acre-ft.
* Total water used for irrigation in July cannot exceed 18,000 acre-ft.
* Total water used for irrigation in August cannot exceed 6,000 acre-ft.

*See the constraint matrix ‘A’ below.*

**Objective Function**

*Such that:*

**Dual Formulation**

**Decision Variables**

* YLand: Value ($) per acre of land
* YJune: Value ($) per acre-ft of water irrigated in June.
* YJuly: Value ($) per acre-ft of water irrigated in July.
* YAugust: Value ($) per acre-ft of water irrigated in August.

**Constraints**

* Value per acre of hay must equal or exceed the sum of the unit value per acre of land, twice the unit value per acre-ft of water in June, the unit value per acre-ft of water in July, and the unit value per acre-ft of water in August.
* Value per acre of grain must equal or exceed the sum of the unit value per acre of land, the unit value per acre-ft of water in June, and twice the unit value per acre-ft of water in July.

*See constraint matrix ‘AT’ below.*

**Objective Function**

*Such that:*

# Appendix B: Problem 7.1 Formulation

Figure 1. Problem schematic and information from text (Bishop, Hughes and McKee 1999)

Diagram

Description automatically generated

Table . Costs of infrastructure installation options

|  |  |  |  |
| --- | --- | --- | --- |
| **Infrastructure** | **Operating Cost** | **Capital Cost** | **Capacity** |
| No Reservoir | $0 | $0 | 0 ac-ft |
| Low Reservoir | $0 | $6,000/year | 300 ac-ft |
| High Reservoir | $0 | $10,000/year | 700 ac-ft |
| Pump | $20/ac-ft | $8,000/year | 2.2 ac-ft/acre |

**Dimensions**

* L = storage or flow locations within the system
  + Reservoir storage (“res”)
  + Diversion to farm from reservoir location (“far”)
  + Release to river from reservoir location (“riv”)
  + Extracted volume from river at pump location (“pmp”)
* T = time in season (t1 “Season 1”, t2 “Season 2”)
* S = size of infrastructure project (nb “No-Build”, s1 “Season 1”, s2 “Season 2”)

**Decision Variables**

* I(S) = binary decision to build reservoir of size S (1=yes, 0=no)
* J = binary decision to install a pump
* X(L, T) = water allocations (ac-ft) by location L and season T
* Area = irrigated acreage on the farm; same area across all seasons modeled.

**Objective Function**

Maximize net benefits. Net benefits are the benefits from irrigating acreage in both time steps minus capital and operating costs.

**Constraints (GAMS Code included below description)**

* Reservoir storage less than or equal to built capacity

X("res", t) SUM(s, R\_Cap(s)\*I(s));

* Mass balance at reservoir in each time step.

Zero initial storage in Time step one

X("res", "t1") Inflow("t1") + 0 - X("riv", "t1") - X("far", "t1");

In time step #2

X("res", "t2") =E= Inflow("t2") + X("res", "t1") - X("riv", "t2")-X("far", "t2")

* Area irrigated by water supplies

A (X("far", t) + X("pmp", t))/I\_Dem(t) t

* Non-negative flow and storage variables
* Mass balance on flow in river for each season

X("riv", t) + GW\_DRAIN\*365/2 - X("pmp", t) 0 t

* Only build one reservoir

SUM(s,I(s)) = 1

* Pump use within pump capacity for all time steps

X("pmp", t) J \* P\_Cap \* 365/2 t

See [GAMS code and listing file](https://github.com/joshuatward/JTW_CEE5410_Repo/tree/master/HW5%20Dual%20and%20Integer) for complete variable and parameter definitions (Ward 2020).

*Note: This sheet was adapted from in-class materials provided on October 2, 2020, by Dr. David Rosenberg* (Rosenberg, Lecture: In-class Integer Programming Model Formulation 2020)*.*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Category**  **(Max. Score)** | **No Evidence** | **Doesn’t Meet Standard** | **Nearly Meets Standard** | **Meets Standard** | **Exceeds Standard** | **Self- Score** | **Instructor Score** |
| **Title**  **(1)** | Absent  0 | Evidence of two or less  0 | Evidence of three  0 | Evidence of four  1 | Title – can assess main point from title alone; Name, Instructors’ Names, Course, Date, Neatly finished 1 | 1 |  |
| **Introduction**  **(3)** | Absent, no evidence  0 | There is no clear introduction or main topic.  1 | Introduction states the main topic but either:   1. Does not give a full overview, Or: 2. Too detailed, leading to annoying repetition later. 2 | The introduction states the main topic and previews the structure of the report.  2 | The introduction states the main topic and previews the structure of the report. Good overview of the problem and solution approach. Gives enough detail to motivate the reader to continue reading.  3 | 3 |  |
| **Organization and structural development of the idea: procedure, results, conclusions**  **(10)** | No content provided.  0 | Paragraphs fail to develop the main idea. No section headers or guide to help the reader understand how material is organized.  1 – 5 | Organization of ideas not fully developed. Paragraphs lack supporting detail sentences. No transitions and/or ineffective section headers.  6 - 7 | Paragraph development present but not perfected. Each paragraph has sufficient supporting detail sentences. Few transitions.  8 | Writer demonstrates logic and sequencing of ideas through well-developed section headers, paragraphs, and transitions. The first sentence of each paragraph is the summary sentence.  9 - 10 | 10 |  |
| **Technical Correctness**  **(70)** | Questions not addressed.  3 – 42% | The writer has no clue what they are talking about.  45 – 58% | Sketchy: left out required design points. Did not work on this as much as you should have, and it shows. Many important answers are incorrect.  61 – 79% | Discussion lacks adequate detail, but all the necessary points are covered and nearly all answers are correct.  82 – 88% | Provides what was explicitly asked for. The function of each piece is demonstrated to the reader in adequate, but not overwhelming, detail. Answers are correct and reasonable.  91 – 100% |  |  |
| 1. Dual problem | | | | |  |  |
| 1. Primal and Dual model formulations (15) | | | | | 15 |  |
| 1. Primal and Dual model solutions [objective function, decision variable, shadow value, and reduced cost values] (10) | | | | | 10 |  |
| 1. Comparison and interpret solutions (10) | | | | | 9 |  |
| 1. Integer programming | | | | |  |  |
| 1. Problem formulation (decision variables [5], objective function [5], constraints [5]) | | | | | 5 |  |
| 1. Solution method (10) | | | | | 9 |  |
| 1. Results and recommendations (10) | | | | | 10 |  |
| **Category**  **(Max. Score)** | **No Evidence** | **Doesn’t Meet Standard** | **Nearly Meets Standard** | **Meets Standard** | **Exceeds Standard** | **Self- Score** | **Instructor Score** |
| **Word Usage and Format**  **(10)** | Not applicable | Numerous and distracting errors in punctuation, capitalization, spelling, sentence structure, word usage, significant figures, tables, and figures. Data vomited onto page(s). Unacceptable / unprofessional at the graduate level. 1 – 5 | Misspelled words, poor English grammar and word choice. Main body of report is either longer or significantly less than one page. Figures are too small and/or under-labeled, although they are usually of acceptable quality and focus. Tables incoherent or not cohesive. Bad font sizes. Too much or too little data in appendices. Could be improved by being more meticulous.  6 - 7 | Almost no errors in punctuation, capitalization, spelling, sentence structure, word usage, significant figures, and presentation of figures, tables, and appendices. Main body of report is one page or less  8 | Punctuation, capitalization, spelling, sentence structure, word usage, and significant figures all correct. Main body of report is one page or less. Clear, consistent fonts. Good word processing skills. Figures have adequate contrast. Informative figure and table titles and legends. Figures have appropriate axis tick spacing, labels, units, and legends. Table columns cohesive, labeled, and specify units. Document is stapled. Appendices, if provided, are separated by topic, and each have a title, discussion, and proper formatting and display of information 9 - 10 | 10 |  |
| **Conclusion**  **(4)** | Absent  0 | Incomplete and/or not focused. 1 | The conclusion does not adequately restate the main results. 2 | The conclusion restates the main results. 3 | The conclusion restates the main results, and is an effective summary. 4 | 4 |  |
| **References**  **(0)** | Absent  0 | Numerous errors, off-the-wall sources used. 0 | Some errors in citing format; more sources should be cited.  1 | Prior work cited with few errors.  2 | All prior work and data sources are cited in the correct format with no errors.  2 | 0 |  |
| **TOTAL** (98) | 96 – (whatever is deducted for lateness) | | | | | See 🡨 |  |

COMMENTS: I spent between 4-5 hours on 10/4/2020 working the dual solution to HW4 instead of HW3 and ended up using enough time that I couldn’t get this assignment finished on time and still turn in quality work. I was scheduled to work an additional 3.5 hours on 10/5 on top of class and meetings, which further set me behind. I understand if points must be deducted for tardiness, though I would appreciate any mercy you can have on my score. I worked hard to turn in quality work even though it was late. Thank you.